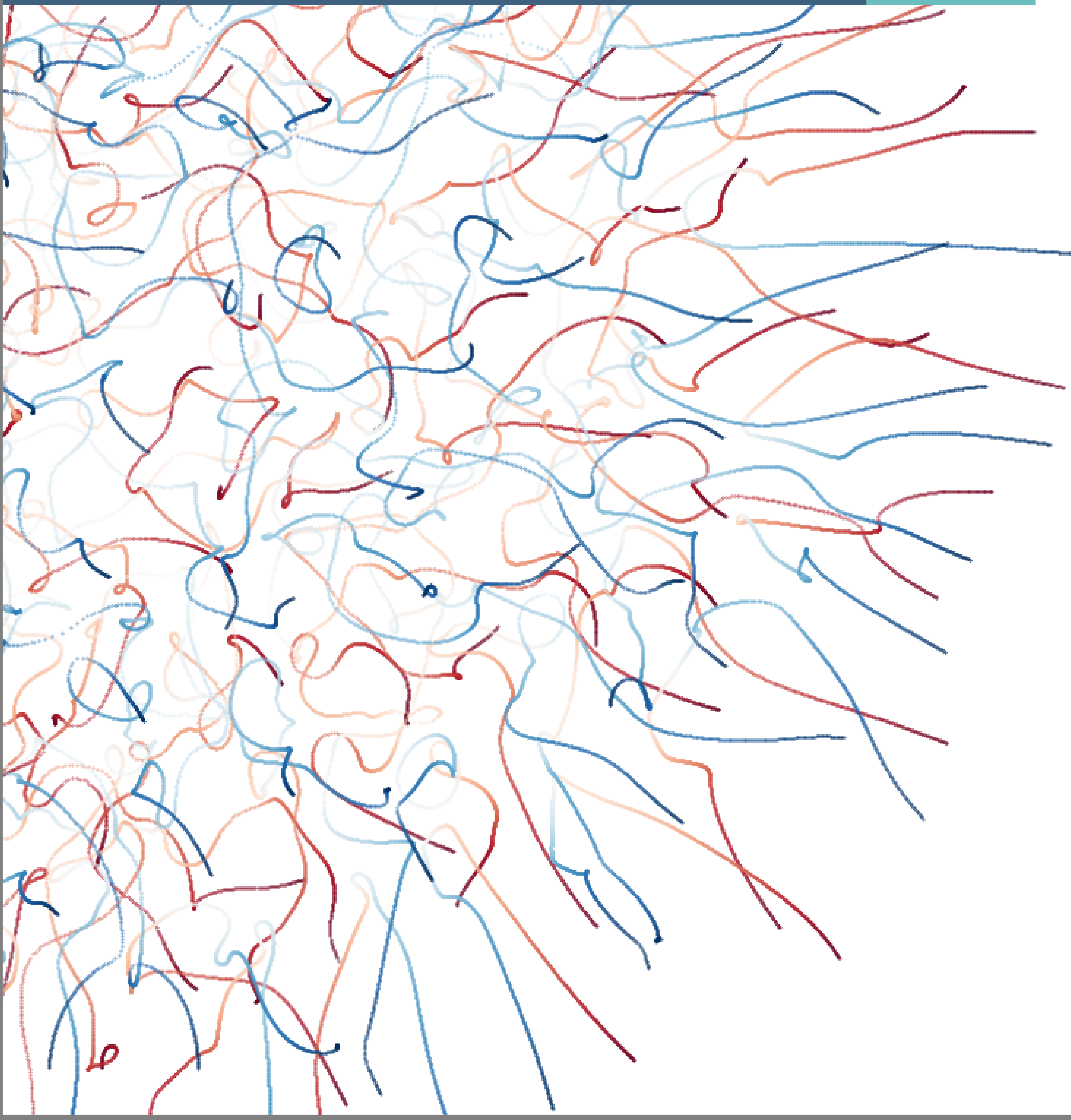
Asymptotics Randomness Nonlinearity and Orthogonality

Book of Abstracts

May 27–31, 2024

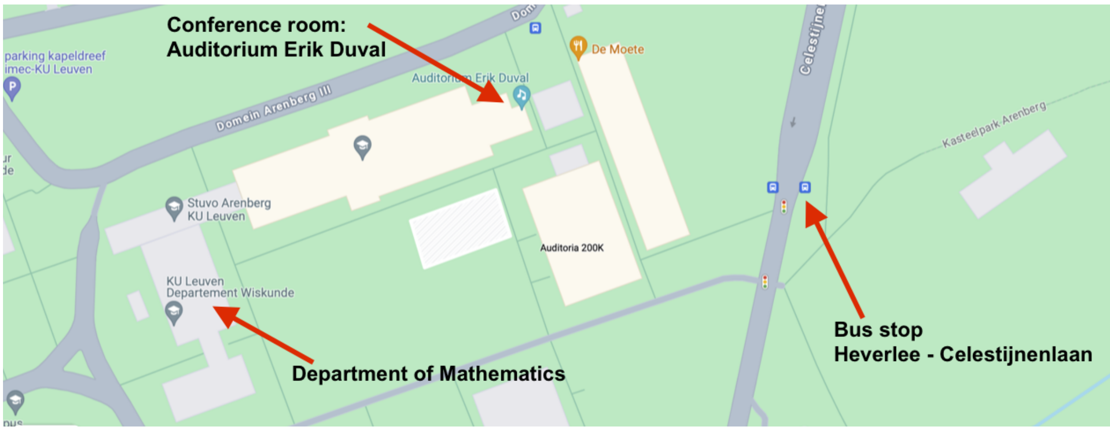
Leuven, Belgium



ARNO24 - A conference in celebration of Arno Kuijlaars' 60th birthday.

Detailed Program

The talks will take place in room 200A 00.225 (auditorium Erik Duval), in the Department of Computer Science (Celestijnenlaan 200A, Heverlee), which is next to the Department of Mathematics.



Monday 27 May

8:00–9:30 Registration + welcome coffee

9:30–10:00 Opening ceremony

10:00–10:50 Ken McLaughlin (Tulane University)
Riemann Hilbert analysis, $\overline{\partial}$ methods, and asymptotic analysis of integrable nonlinear PDEs

10:50–11:20 Dan Dai (City University of Hong Kong)
On the eigenvalue rigidity of the Jacobi unitary ensemble

11:20–12:10 Mylène Maïda (Université de Lille)
Random partitions and topological expansion of 2D Yang-Mills partition function

12:10–14:00 Lunch

14:00-14:50 Thomas Bothner (University of Bristol)
Universality for random matrices with an edge spectrum singularity

14:50–15:20 Wenkui Liu (KTH Royal Institute of Technology)
Mesoscopic Universality of Orthonormal Polynomial Ensembles: Edge Fluctuations

15:20–16:00 Coffee Break

16:00–16:50 Gernot Akemann (Bielefeld University and University of Bristol)
Random point processes in the plane and applications to ecology

16:50–17:20 Roozbeh Gharakhloo (University of California, Santa Cruz)
Asymptotics and applications of structural deformations of Toeplitz and Hankel determinants

Evening Reception at the Arenberg Castle

Tuesday 28 May

9:00–9:50 Lun Zhang (Fudan University)
Large gap asymptotics of the tacnode process

9:50–10:20 Benjamin Eichinger (TU Wien)
Universality limits with a limit cycle

10:20–11:00 Coffee break

11:00–11:30 Lu Wei (Texas Tech University)
Cumulant structures of entanglement entropy

11:30–12:20 Sunil Chhita (Durham University)
The two-periodic Aztec diamond

12:20–14:00 Lunch

14:00–14:50 Kurt Johansson (KTH)
Coulomb gas and the Grunsky operator on a Jordan domain with corners

14:50–15:20 Mateusz Piorkowski (KU Leuven)
Matrix-valued orthogonal polynomials and Wiener-Hopf factorizations

15:20–16:00 Coffee break

16:00–16:50 Alice Guionnet (CNRS and Ecole Normale Supérieure Lyon)
Discrete Beta-ensembles and the uses of Nekrasov's equations

16:50–17:20 Roger Van Peski (KTH Royal Institute of Technology)
New limits in discrete random matrix theory

Wednesday 29 May

9:00–9:50 Marco Bertola (Concordia University)
Potential theory on elliptic curves and applications to asymptotic questions

9:50–10:20 Amílcar Branquinho (Universidade de Coimbra)
Spectral theory for bounded banded matrices and mixed multiple orthogonal polynomials

10:20–11:00 Coffee break

11:00–11:30 Thorsten Neuschel (Dublin City University)
Boundary Asymptotics of Non-Intersecting Brownian Motions: Pearcey, Airy and a Transition

11:30–12:20 Tamara Grava (University of Bristol and SISSA)
Soliton gas for the focusing nonlinear Schrödinger equation

12:20–12:50 Bingying Lu (SISSA)
Soliton Gas for Toda Lattice

12:50–14:00 Lunch

Free afternoon

Thursday 30 May

9:00–9:50 Aleksey Bufetov (Leipzig University)
Domino tilings of the Aztec diamond and Schur generating functions

9:50–10:20 Kailun Chen (Institute of Mathematics, Leipzig University)
Mallows product measure

10:20–11:00 Coffee break

11:00–11:30 Thomas Chouteau (Universidade Sao Paulo)
Integro-differential Painlevé II hierarchy in relation with Gelfand-Dickey hierarchy

11:30–12:20 Vadim Gorin (UC Berkeley)
Boundary limits for the six-vertex model via orthogonal polynomials

12:20–14:00 Lunch

14:00–14:50 Pavel Bleher (Indiana University Purdue University Indianapolis)
Topological Expansion and Phase Diagram for Ensembles of Random Matrices with Complex Potentials

14:50–15:20 Kenta Miyahara (Indiana University Purdue University Indianapolis)
Connection formulae for the radial Toda equations

15:20–16:00 Coffee break

16:00–16:30 Nathan Hayford (KTH)
The Ising Model Coupled to 2D Gravity I: Genus 0 Partition Function

16:30–17:20 Peter Miller (University of Michigan)
Universality in the Small-Dispersion Limit of the Benjamin-Ono Equation

Evening Conference Dinner at the restaurant and brewery Domus

Friday 31 May

9:00-9:50 Andrei Martínez-Finkelshtein (Baylor University)
Classical multiple orthogonal polynomials and free probability

9:50–10:20 Olof Rubin (Lund University)
Chebyshev polynomials on polynomial preimage sets

10:20–11:00 Coffee break

11:00–11:30 Sergey Berezin (KU Leuven)
Convergence of Dynamical Additive Statistics to Gaussian Processes

11:30–12:00 Grzegorz Świderski (Polish Academy of Sciences)
Limit theorems for global linear statistics of some Orthogonal Polynomial Ensembles

12:30–14:00 Lunch (french fries on campus)

14:00–14:30 Matthias Allard (KU Leuven and University of Melbourne)
Correlation functions between singular values and eigenvalues

14:30–15:20 TBA

15:20 Closing ceremony

Poster Session

The posters will be displayed during the coffee breaks.

Victor Julio Alves (Universidade de São Paulo)
Lifting of Jacobi-like polynomials to genus 1 compact Riemann Surfaces

Benedikt Buchecker (TU Wien)
Szegő's theorem for Jordan arcs

Juan Díaz (Universidade de Aveiro)
Classical Multiple Orthogonal Polynomials of Type I

Giulio Ruzza (Universidade de Lisboa)
Determinantal point processes and integrable PDEs

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John Jairo López Santander (Tulane University)
Asymptotics and Zeros of a special family of Jacobi Polynomials

Charles Ferreira dos Santos (Universidade de São Paulo)
Asymptotics for gap probabilities of a conditional thinning of the Airy2 point process

Abstracts

Random point processes in the plane and applications to ecology

Gernot Akemann

Bielefeld University and University of Bristol

Random point processes including determinantal ones are popular models in ecology. In this talk I will put the two-dimensional Coulomb gas at general inverse temperature $\beta \geq 0$ in a such a perspective. Away from the integrable point $\beta = 2$, corresponding to the complex Ginibre ensemble, and the Poisson limit at $\beta = 0$, very little is known about the local statistics. We therefore resort to numerical simulations to determine the nearest and next-to nearest spacing to model data from biology. An alternative, approximate description is based on a 2×2 beta ensemble. Annual ensembles of nests of three different birds of prey in the area of the Teutoburger Wald close to Bielefeld will be modelled by such a simple random point process, in fitting an effective beta to the data. In such a way different repulsion strengths can be quantified, comparing the inter and intra-species repulsion, as well as their change over time. This is joint work with Adam Mielke, Patricia Paessler and the group of Oliver Krueger

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Correlation functions between singular values and eigenvalues

Matthias Allard

KU Leuven and University of Melbourne

Exploiting the explicit bijection between the density of singular values and the density of eigenvalues for bi-unitarily invariant complex random matrix ensembles of finite matrix size we aim at finding the induced probability measure on j eigenvalues and k singular values that we coin j,k-point correlation measure. We fully derive all j,k-point correlation measures in the simplest cases for matrices of size $n = 1$ and $n = 2$. For $n > 2$, we find a general formula for the $1,1$-point correlation measure. This formula reduces drastically when assuming the singular values are drawn from a polynomial ensemble, yielding an explicit formula in terms of the kernel corresponding to the singular value statistics. These expressions simplify even further when the singular values are drawn from a Pólya ensemble and extend known results between the eigenvalue and singular value statistics of the corresponding bi-unitarily invariant ensemble.

Convergence of Dynamical Additive Statistics to Gaussian Processes

Sergey Berezin
KU Leuven

The Mittag–Leffler ensemble, when restricted to an origin-centered disk of radius ρ inside the droplet, possesses a natural hard edge at the scale of O(1/n). Take a bounded measurable function φ. We consider the additive statistic defined by $\varphi_t(z) = \varphi(z) \cdot 1[|z| > t]$, $t \in [0, \rho]$. The statistic evolves with time t, forming a continuous-time stochastic process on $[0, \rho]$. As the number of particles increases, this process converges in the appropriate functional space to a Gaussian process. Our result extends the finite-dimensional CLT for $\varphi = 1$, previously established by Y. Ameur, C. Charlier, J. Cronvall, and J. Lenells (2022). Furthermore, our approach allows for studying first hitting times of dynamical additive statistics, resulting in another functional central limit theorem.

Potential theory on elliptic curves and applications to asymptotic questions

Marco Bertola
Concordia University

In this talk I will review recent work in collaboration with Arno and Groot on the extension of the familiar setup of potential theory used pervasively in the asymptotic analysis of orthogonal polynomials. The extension involves considering the same problem on a Riemann surface of higher genus (in concrete term, an elliptic curve).

A particular emphasis that I want to place is on the potential (ah!) applications of these techniques to the asymptotic analysis of "orthogonal functions" on Riemann surfaces and related Padé—like approximation schemes on higher genus Riemann surfaces, especially in the "scaling weight" type of problems.

Topological Expansion and Phase Diagram for Ensembles of Random Matrices with Complex Potentials

Pavel Bleher

Indiana University Purdue University Indianapolis

We will discuss recent rigorous results on the topological expansion and phase diagram in ensembles of random matrices with complex cubic and quartic potentials. The proofs are based on the Riemann—Hilbert approach to semiclassical asymptotics of non-Hermitian orthogonal polynomials and on the theory of S-curves and quadratic differentials.

This is an ongoing project with Ahmad Barhoumi, Marco Bertola, Alfredo Deaño, Roozbeh Gharakhloo, Ken McLaughlin, Alex Tovbis, and Maxim Yattselev.

Universality for random matrices with an edge spectrum singularity

Thomas Bothner

University of Bristol

We study invariant random matrix ensembles defined on complex Hermitian matrices with a single root type singularity and one-cut regular density of states. Assuming that the singularity lies within the soft edge boundary layer we compute asymptotics of the model's generating functional by using Riemann-Hilbert problems for orthogonal polynomials and integrable operators. This extends an old result by Forrester and Witte and is based on ongoing joint work with Toby Shepherd (Bristol). The contributions of Arno to our problem will be emphasized throughout.

Spectral theory for bounded banded matrices and mixed multiple orthogonal polynomials

Amílcar Branquinho

Universidade de Coimbra

Banded bounded matrices, which represents non normal operadors, of oscilatory type that admit a positive bidiagonal factorization are considered. To motivate the relevance of the oscillatory character the Favard theorem for Jacobi matrices is revisited and it is shown that after an adequate shift of the Jacobi matrix one gets an oscillatory matrix. In this work we present a spectral theorem for this type of operators and show how the theory of multiple orthogonal polynomials apply. This is joint work with Ana Foulquié (U Aveiro) and Manuel Mañas (U Complutense Madrid).

Szegő's theorem for Jordan arcs

Benedikt Buchecker
TU Wien

The n-th Christoffel function, $\lambda_n(\mu, z_0)$ for a point $z_0 \in \mathbb{C}$ and a finite measure μ is

$$\lambda_n(\mu, z_0) = \inf \left\{ \int |P|^2 d\mu \mid P \text{ is a polynomial of degree at most } n \text{ and } P(z_0) = 1 \right\}.$$

It is natural to extend this notion to $z_0 = \infty$ and define $\lambda_n(\mu, \infty)$ to be the square of the norm of the monic orthogonal polynomials associated to μ. The classical Szegő theorem provides a full asymptotic description of $\lambda_n(\mu, z_0)$ for $|z_0| > 1$ and $z_0 = \infty$ and arbitrary finite measures supported on the unit circle. Widom has proved a version of Szegő's theorem for measures supported on C^{2+} Jordan arcs for the point $z_0 = \infty$ and purely absolutely continuous measures belonging to the Szegő class. We extend this result by providing a full analog of Szegő's theorem for finite measures supported on a C^{2+} Jordan arc Γ. That is, we give explicit asympotitcs of $\lambda_n(\mu, z_0)$ for arbitrary finite measures supported on Γ, (i.e., we do not assume absolute continuity), and for all points $z_0 \in \mathbb{C} \cup \{\infty\} \setminus \Gamma$. Moreover, if the measure is in the Szegő class, we provide explicit asymptotics for the extremal and orthogonal polynomials.

Domino tilings of the Aztec diamond and Schur generating functions

Aleksey Bufetov
Leipzig University

Random domino tilings of the Aztec diamond can be studied in various (related) ways, including determinantal processes, orthogonal polynomials, variational principle, ... In this talk I will discuss some old and new results about them that can be obtained via the technique of Schur generating functions.

Mallows product measure

Kailun Chen

Institute of Mathematics, Leipzig University

Q-exchangeable ergodic distributions on the infinite symmetric group were classified by Gnedin-Olshanski (2012). In this paper, we study a specific linear combination of the ergodic measures and call it the Mallows product measure. From a particle system perspective, the Mallows product measure is a reversible stationary blocking measure of the infinite-species ASEP and it is a natural multi-species extension of the Bernoulli product blocking measures of the one-species ASEP. Moreover, the Mallows product measure can be viewed as the universal product blocking measure of interacting particle systems coming from random walks on Hecke algebras. For the random infinite permutation distributed according to the Mallows product measure we have computed the joint distribution of its neighboring displacements, as well as several other observables. The key feature of the obtained formulas is their remarkably simple product structure. We project these formulas to ASEP with finitely many species, which in particular recovers a recent result of Adams-Balazs-Jay, and also to ASEP(q,M). Our main tools are results of Gnedin-Olshanski about ergodic Mallows measures and shift-invariance symmetries of the stochastic colored six vertex model discovered by Borodin-Gorin-Wheeler and Galashin.

The two-periodic Aztec diamond

Sunil Chhita
Durham University

The two-periodic Aztec diamond is a random tiling model which features three macroscopic regions known as frozen, rough and smooth, which are each characterized by their decay of correlations. In this talk, we will introduce the model as well as describe the behavior at the rough-smooth boundary. This talk is based on joint work with Duncan Dauvergne and Thomas Finn.

Integro-differential Painlevé II hierarchy in relation with Gelfand-Dickey hierarchy

Thomas Chouteau

Universidade Sao Paulo

In this presentation, I will introduce a new construction of the integro-differential Painlevé ii hierarchy through the study of the Airy kernel at finite temperature of higher order. This result will be based on the investigation of a $(2n \times 2n)$ sized Riemann-Hilbert problem associated with the operator of the higher-order Airy kernel. Among other things, we will deduce from this Riemann-Hilbert problem a Lax pair for our system, enabling us to obtain equations of the Gelfand-Dickey hierarchy. Furthermore, we will establish the connection between the multiplicative statistics of the process of the higher-order Airy kernel at finite temperature and the integro-differential hierarchy of Painlevé II. This presentation is based on ongoing work.

On the eigenvalue rigidity of the Jacobi unitary ensemble

Dan Dai

City University of Hong Kong

We consider rigidity estimates for the eigenvalues of the Jacobi unitary ensemble. Following ideas in Claeys et al. (Duke Math. J., 2021), our investigation focuses on the extreme values of the associated log-correlated stochastic processes, and asymptotics of large Hankel determinants with various singularities. Through these analyses, we achieve sharp upper and lower bounds for the eigenvalue fluctuation.

Classical Multiple Orthogonal Polynomials of Type I

Juan Díaz

Universidade de Aveiro

Multiple orthogonal polynomials are a generalization of usual orthogonal polynomials that arises from considering orthogonality respect to, not one, but an arbitrary number of weight functions. This leads to two different kinds of polynomials. On one hand there are the multiple type II polynomials, that have been widely studied and many families are already known. On the other hand there are the multiple type I polynomials. These ones have been less studied and not so many families are known. Here are given explicit expressions for the type I polynomials corresponding to the classical families of Jacobi–Piñeiro, Laguerre and Hermite. All of them were unknown so far despite the corresponding type II polynomials for these families being known from twenty, or even more, years ago. Finally, all of them are expressed through special functions such as the generalized hypergeometric series or the Kampé de Fériet series.

Universality limits with a limit cycle

Benjamin Eichinger
TU Wien

In this talk I will survey recent advances in the study of universality limits of orthogonal polynomials. I will discuss the case that the Christoffel–Darboux kernel has a regularly varying rescaling limit. These types of universality limits typically appear at the bulk or the edge of the spectrum. We show that at accumulation points of spectral gaps the scaling behavior might be quite different. We will discuss rescaling limits, where there is not a unique limit kernel but a full limit cycle. We show that balanced measures on a real Julia set of an arbitrary expanding polynomial provide natural examples for such type of rescaling limits.

This talk is based on joint works with Milivoje Lukić, Brian Simanek, Harald Woracek and Peter Yuditskii.

Asymptotics for gap probabilities of a conditional thinning of the Airy2 point process

Charles Ferreira dos Santos

Universidade de São Paulo

The Airy_2 point process is a determinantal point process that appears as a limit process for eigenvalues of hermitian random matrices. Here we will consider a conditional thinning of the Airy_2 process, whose correlation kernel is built upon a particular solution of a nonlinear Schrödinger equation. More recently, a connection was found with the solutions to the KPZ equation

Our goal is to provide asymptotics for a gap probability, which here means the probability of the largest particle being no greater than τ when $\tau \to \pm\infty$. This gap probability coincides with the Fredholm determinant of an integral operator with IIKS-integrable kernel. This, in its turn, is related to the solution of a matrix Riemann-Hilbert problem (RHP), being amenable to the use of nonlinear steepest descent method for RHPs.

This is a joint work with Guilherme Silva (ICMC/USP São Carlos) and Lun Zhang (Fudan University). The author is funded by PRIP/USP.

Asymptotics and applications of structural deformations of Toeplitz and Hankel determinants

Roozbeh Gharakhloo

University of California, Santa Cruz

It is well-known that Toeplitz and Hankel determinants characterize many fundamental objects in engineering, physics, and mathematics. Motivated by multiple applications across several areas, there has been a recent growing interest in studying the structural deformations of Toeplitz and Hankel determinants. Among those are Toeplitz+Hankel, bordered Toeplitz/Hankel, framed Toeplitz/Hankel, and slant Toeplitz/Hankel[1] determinants. These structurally deformed determinants appear in many different contexts such as the Ising model, the six-vertex model, random matrix models, quantum spin chains, ensembles of non-intersecting paths and more. After a brief introduction and going over some of the applications, I will discuss some recent developments in the Riemann-Hilbert approach to the asymptotics of some of these determinants.

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[1]whose jk-entry is the $(pj \pm qk)$-th Fourier coefficient of a symbol, for a fixed $p, q \in \mathbb{N}$. The choice $(p, q) = (1, 1)$ reduces these to the standard Toeplitz and Hankel structures.

Boundary limits for the six-vertex model via orthogonal polynomials

Vadim Gorin

UC Berkeley

Take a random configuration of (a, b, c)-weighted six-vertex model in a very large planar domain. What does it look like near a straight segment of the boundary? We investigate this question on the example of the model in $N * N$ square with Domain Wall Boundary Conditions and find that the answer depends on the value of $\Delta = (a^2 + b^2 - c^2)/(2ab)$: there is a single universal limiting object for all $\Delta < 1$ and a richer class of limits at $\Delta > 1$. Important roles in our analysis are played by a reduction to a log-gas system and RH-based analysis of the associated orthogonal polynomials.

Soliton gas for the focusing nonlinear Schrödinger equation

Tamara Grava

University of Bristol and SISSA

We consider a gas of random N solitons for the focusing nonlinear Schrödinger equation and we study the limit as N goes to infinity. We derive the limiting solution and we prove that its fluctuations are Gaussian random variables.

Discrete Beta-ensembles and the uses of Nekrasov's equations

Alice Guionnet

CNRS and Ecole Normale Supérieure Lyon

NA

The Ising Model Coupled to 2D Gravity I: Genus 0 Partition Function

Nathan Hayford
KTH

The 2D Ising model is one of the most celebrated examples of an exactly solvable lattice model. Motivated by problems in statistical mechanics and 2D quantum gravity, in 1986 Vladimir Kazakov considered the Ising model on a random planar lattice using techniques from random matrix theory. He was able to derive a formula for the free energy of this model, and made the first prediction of the Kniznik-Polyakov-Zamolodchikov (KPZ) formula for the shift of the critical exponents of a conformal field theory when coupled to quantum gravity. Unfortunately, his derivation was not mathematically rigorous, and the formula he obtained for the free energy was somewhat unwieldy. In this talk, I will review some of the details regarding both the Ising model and random matrices, and sketch a rigorous proof of Kazakov's formula for the free energy. I will also present a parametric formula for the free energy, which seems to be new. This is joint work with Maurice Duits and Seung-Yeop Lee.

Asymptotics and Zeros of a special family of Jacobi Polynomials

John Jairo López Santander

Tulane University

Classical Jacobi polynomials, $p_n(x) = p_n^{(\alpha,\beta)}(x)$, are well-known for their orthogonality on the interval $[-1, 1]$, with respect to the weight function $w(x; \alpha, \beta) = (1 - x)^\alpha (1 + x)^\beta$, where $\alpha, \beta > -1$. By analytic continuation on the parameters α, β, the polynomials can be studied for general complex parameters. However, when $Re(\alpha) \leq -1$ or $Re(\beta) \leq -1$, the classical orthogonality property on $[-1, 1]$ no longer holds and consequently, the zeros may not be real.

In this poster, we will present the asymptotic analysis of a specific family of Jacobi polynomials with non-classical parameters of the form $\alpha_m = m + 1/2$, $\beta_m = -m - 1/2$. These polynomials arise in connection with the evaluation of integrals by Boros and Moll in 1999. We derive global asymptotics and the location of their zeros from the corresponding Riemann Hilbert problem. In particular, we prove that the zeros accumulate on the left branch of the curve $|1 - z^2| = 1$. Also, particularly interesting is the behavior near the origin, where local asymptotics are described in terms of the parabolic cylinder function.

It is worth highlighting that this investigation corresponds to a limiting case in the works from Kuijlaars et al. (2000, 2004, 2005), thus extending our understanding of Jacobi polynomials in the non-classical parameter regimes.

Coulomb gas and the Grunsky operator on a Jordan domain with corners

Kurt Johansson

KTH

I will discuss the asymptotics of the partition function of a Coulomb gas in the plane confined to a region bounded by a Jordan curve with corners. The analysis is based on an exact formula that expresses the partition function as a finite Fredholm determinant of an operator coming from the Grunsky operator for the curve. From this formula it follows immediately that the suitably normalized partition function has a limit if and only if the Jordan curve is a so called Weil-Petersson quasicircle. If the curve has corners it is no longer a Weil-Petersson quasi circle and the the suitably normalized partition function diverges like a power of n, the number of particles, and the exponent depends on the angles in an interesting way.

Lifting of Jacobi-like polynomials to genus 1 compact Riemann Surfaces

Victor Julio Alves

Universidade de São Paulo

In recent years Marco Bertola presented a construction for higher genus orthogonal “sections”, meromorphic functions defined on a higher genus Riemann surface that satisfies an orthogonality property akin to orthogonal polynomials on the complex plane. We present a brief introduction to the genus one case, that we call elliptic orthogonal polynomials, and a decomposition theorem in terms of two families of Jacobi-like orthogonal polynomials on the real line. Some algebraic properties and open questions are also presented.

Mesoscopic Universality of Orthonormal Polynomial Ensembles: Edge Fluctuations

Wenkui Liu

KTH Royal Institute of Technology

In this talk, I am going to report on ongoing work on the fluctuations of mesoscopic linear statistics for orthogonal polynomial ensembles at the edges. We show that this behaviour is universal in the sense that if the recurrence coefficients are slowly varying, the asymptotic mesoscopic fluctuations are expected to be the same. The rate of varying controls the range of the scales on which the linear statistics probes. Our main tool is an analysis of the resolvent for the associated Jacobi matrices. To this end, we improve the Combes-Thomas estimation for the Jacobi matrices for slowly varying entries. As a particular consequence, we obtain a mesoscopic central limit theorem near the soft edge for Unitary Ensembles with varying exponential weight with strictly convex smooth potentials. An example of the hard edge is that we can show the Laguerre and modified Jacobi Unitary Ensembles on all mesoscopic scales.

Soliton Gas for Toda Lattice

Bingying Lu
SISSA

We analytically study the long time asymptotics of a class of solution that describes then N-soliton gas, where $N \to \infty$. We formulate this problem by a Riemann-Hilbert problem that we can rigorously analyze. Using the soliton gas formulation, we study the dynamics of the solution with step-like initial data. Dispersive shock or rarefraction region ensues, depending on the initial step, and in long time the solution decomposes to three different regions, where their asymptotics are described with the Riemann-Hilbert problem.

Random partitions and topological expansion of 2D Yang-Mills partition function

Mylène Maïda

Université de Lille

In the fifties, Chen Ning Yang and Robert Mills made a major breakthrough in quantum field theory by extending the concept of gauge theory to non-abelian groups. Since then, the study of Yang-Mills theory has been a very active field of research both in mathematics and physics. In particular, in the last two decades, significant progress has been made on the rigorous mathematical understanding of the theory on two-dimensional manifolds with gauge group U(N) or SU(N), and of their limit as N grows to infinity. If this talk, I will give an overview of some of these results. I will also explain in more details how the probabilistic study of well chosen random partitions allows us to give rigorous proofs of some topological expansions of the partition function predicted by physicists Gross and Taylor in the nineties. This is joint work with Thibaut Lemoine (Université de Lille).

Classical multiple orthogonal polynomials and free probability

Andrei Martínez-Finkelshtein
Baylor University

The concept of finite free convolution of polynomials arises within the framework of free probability theory. Recently, it has gained attention due to its applications in the study of hypergeometric polynomials. Specifically, these polynomials can be represented as a finite free convolution of more elementary building blocks. This representation, combined with the preservation of real zeros and interlacing properties through free convolutions, provides an effective tool for analyzing when all roots of a particular hypergeometric polynomial are real. Consequently, this approach offers a fresh perspective on the zero properties of hypergeometric polynomials. Furthermore, this representation remains valid even in the asymptotic regime, allowing us to express the limit zero distribution of generalized hypergeometric polynomials as a free convolution of more "elementary" measures. We demonstrate these results through applications to some families of multiple orthogonal polynomials. This is a joint work with R. Morales (Baylor University) and Daniel Perales (Texas A&M University).

Riemann Hilbert analysis, $\overline{\partial}$ methods, and asymptotic analysis of integrable nonlinear PDEs

Ken McLaughlin
Tulane University

I will present examples of asymptotic analysis of integrable systems (broadly interpreted) and explain some of the techniques which have been developed to accomplish this analysis, in many instances originating in work of Arno and his collaborators.

Universality in the Small-Dispersion Limit of the Benjamin-Ono Equation

Peter Miller

University of Michigan

This talk concerns the Benjamin-Ono (BO) equation of internal wave theory, and properties of the solution of the Cauchy initial-value problem in the situation that the initial data is fixed but the coefficient of the nonlocal dispersive term in the equation is allowed to tend to zero (i.e., the zero-dispersion limit). It is well-known that existence of a limit requires the weak topology because high-frequency oscillations appear even though they are not present in the initial data. Physically, this phenomenon corresponds to the generation of a dispersive shock wave. In the setting of the Korteweg-de Vries (KdV) equation, it has been shown that dispersive shock waves exhibit a universal form independent of initial data near the two edges of the dispersive shock wave, and also near the gradient catastrophe point for the inviscid Burgers equation from which the shock wave forms. In this talk, we will present corresponding universality results for the BO equation. These have quite a different character than in the KdV case; while for KdV one has universal wave profiles expressed in terms of solutions of Painlevé-type equations, for BO one instead has expressions in terms of classical Airy functions and Pearcey integrals. These results are proved for general rational initial data using a new approach based on an explicit formula for the solution of the Cauchy problem for BO. This is joint work with Elliot Blackstone, Louise Gassot, Patrick Gérard, and Matthew Mitchell.

Connection formulae for the radial Toda equations

Kenta Miyahara
Indiana University Purdue University Indianapolis

Since the invention of the 1D Toda lattice equation in 1967, many types of Toda equations have been considered. In this presentation, we will talk about the global asymptotic behavior of the radial solutions of the 2D periodic Toda lattice equation of type A_n. The principal issue is the connection formulae between the asymptotic parameters describing the behavior of the general solution at zero and infinity. To reach this goal we use a fusion of the Iwasawa factorization in the loop group theory and the Riemann-Hilbert nonlinear steepest descent method of Deift and Zhou which applies to 2D Toda in view of its Lax integrability. A principal technical challenge is the extension of the nonlinear steepest descent analysis to Riemann-Hilbert problems of matrix rank greater than 2. As a first nontrivial example, we meet this challenge for the case $n = 2$ (the rank 3 case) and it already captures the principal features of the general n case. This is a joint work with Martin Guest, Alexander Its, Maksim Kosmakov, and Ryosuke Odoi.

Boundary Asymptotics of Non-Intersecting Brownian Motions: Pearcey, Airy and a Transition

Thorsten Neuschel

Dublin City University

We study the asymptotic behavior of the correlations of a large number of non-intersecting Brownian motions at the boundary of their spectrum. It is known that universal Pearcey correlations arise close to merging points of the spectrum, whereas universal Airy correlations are found when staying away from merging points. Our analysis shows that at the boundary one more universal limit behavior arises which corresponds to the transition from Pearcey to Airy correlations. This transition is described explicitly in terms of a novel determinantal process. The three cases are distinguished by a remarkably simple integral condition. The results hold under very mild assumptions, in particular we do not require any kind of convergence of the initial con- figurations, and they completely characterise the boundary behavior of non-intersecting Brownian motions for non-vanishing times in the absence of outliers. This is joint work with Martin Venker.

Matrix-valued orthogonal polynomials and Wiener-Hopf factorizations

Mateusz Piorkowski
KU Leuven

This talk explores a family of matrix-valued orthogonal polynomials related to the $k \times l$ periodic Aztec diamond studied by Berggren & Borodin '23. Based on earlier work of Berggren & Duits '19, explicit formulas for these polynomials were obtained recently by Kuijlaars & P. '24. We discuss in detail the peculiar properties of these polynomials and their relation to Wiener-Hopf factorizations. If time permits, we will also discuss the special case of the 2×2 periodic model, for which we were able to express the orthogonal polynomials in terms of Jacobi theta functions.

This work is a collaboration with Arno Kuijlaars.

Chebyshev polynomials on polynomial preimage sets

Olof Rubin

Lund University

The Chebyshev polynomial of degree n corresponding to an infinite compact subset E of the complex plane is the unique nth degree monic polynomial which minimizes the supremum norm on E. In my talk I will discuss new approaches to determine the asymptotics of such polynomials on sets given as preimages under a polynomial P. In particular I will consider lemniscates

$$\{z : |P(z)| \leq r\}$$

and sets of the form

$$\{z : |P(z)| \in [a, b]\}.$$

We will se that such Chebyshev problems can, in certain cases, be related to Chebyshev polynomials on intervals with Jacobi weights. This will allow the determination of norm asymptotics in several cases. One key component is a generalization of a result of Lachance, Saff & Varga on weighted Chebyshev polynomials on the unit circle. The talk is based on joint work together with Jacob S. Christiansen (LU), Alex Bergman (LU) and Benjamin Eichinger (TU Wien).

Determinantal point processes and integrable PDEs

Giulio Ruzza
Universidade de Lisboa

We analyze a number of Fredholm determinants which are relevant in integrable probability (namely, multiplicative expectations of well-known determinantal point processes) from the integrable systems point of view. In particular, we consider integrable PDEs satisfied by such determinants and corresponding asymptotic properties and boundary value problems.

Limit theorems for global linear statistics of some Orthogonal Polynomial Ensembles

Grzegorz Świderski
Polish Academy of Sciences

I will discuss almost sure convergence of properly normalized global linear statistics of Orthogonal Polynomial Ensembles generated by fixed measures with unbounded supports. Nevai's condition from the theory of orthogonal polynomials on the real line turns out to be a useful tool for this task. The talk will be based on arXiv:2404.07566.

Distance in planar maps via orthogonal polynomials

Sofia Tarricone

Institut de Physique Théorique, CEA Saclay

The (integrated) two-point function for planar quadrangulations has been studied from many years in combinatorics. In particular, we focus here on a previous result of Bouttier-Guitter (2010) which describes this generating function as a ratio of Hankel determinants. This result allows us to relate it to the coefficients of the three terms recurrence relation of a family of orthogonal polynomials taken with respect to a specific measure, of "deformed Jacobi type". Standard techniques from orthogonal polynomials should help us to better understand known results about integrability features and asymptotics of this (integrated) two point functions for planas quadrangulations (and perhaps beyond). This presentation is based on work in progress with Jérémie Bouttier.

New limits in discrete random matrix theory

Roger Van Peski
KTH Royal Institute of Technology

Random matrices over the integers and p-adic integers have been studied since the late 1980s as natural models for random groups appearing in number theory, topology and combinatorics. Recently it has also become clear that the theory has close structural parallels with singular values of complex random matrices. I will new universal objects which arise as discrete analogues of the extended sine and Airy processes in classical random matrix theory.

Cumulant structures of entanglement entropy

Lu Wei

Texas Tech University

We will discuss new methods to, in principle, obtain all cumulants of von Neumann entropy over different models of random states. Based on the newly discovered structures of cumulants and the construction of families of related statistics, the new methods are able to circumvent the tedious tasks of simplifying nested summations involving polygamma functions that are ubiquitous when using the existing methods in the literature. This talk is based on an ongoing joint work with Youyi Huang.

Large gap asymptotics of the tacnode process

Lun Zhang

Fudan University

The tacnode process is a universal determinantal point process arising from non-intersecting particle systems and tiling problems. It is the aim of this talk to explore the integrable structure and asymptotics for the gap probability of the thinned/unthinned tacnode process over $(-s, s)$. We establish an integral representation of the gap probability in terms of the Hamiltonian associated with a system of differential equations. With the aids of some remarkable differential identities for the Hamiltonian, we also compute large gap asymptotics, up to and including the constant term in the thinned case. As direct applications, we obtain expectation, variance and a central limit theorem for the associated counting function. Joint work with Luming Yao.
